

Electroweak Monopoles

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We present finite energy analytic monopole and dyon solutions whose size is fixed by the electroweak scale. Our result shows that genuine electroweak monopole and dyon could exist whose mass scale is much smaller than the grand unification scale.

PACS Number(s): 14.80.Hv, 11.15.Tk, 12.15.-y, 02.40.+m

It has generally been believed that in the electroweak theory of Weinberg and Salam there exists no topological monopole of physical interest. The basis for this “non-existence theorem” is, of course, that with the spontaneous symmetry breaking the quotient space $SU(2) \times U(1)/U(1)_{\text{em}}$ allows no non-trivial second homotopy. This belief, however, is unfounded. Indeed recently Cho and Maison [1] have established that the Weinberg-Salam model has exactly the same topological structure as the Georgi-Glashow model, and demonstrated the existence of a new type of monopole and dyon solutions in the standard Weinberg-Salam model. This was based on the observation that the Weinberg-Salam model, with the hypercharge $U(1)$, could be viewed as a gauged CP^1 model in which the (normalized) Higgs doublet plays the role of the CP^1 field. So the Weinberg-Salam model does have exactly the same nontrivial second homotopy as the Georgi-Glashow model which allows topological monopoles. Originally the Cho-Maison solutions were obtained by a numerical integration, but now a mathematically rigorous existence proof has been established which supports the numerical results [2].

The Cho-Maison monopole may be viewed as a hybrid between the Dirac monopole and the 't Hooft-Polyakov monopole, because it has a $U(1)$ point singularity at the center even though the $SU(2)$ part is completely regular. Consequently it carries an infinite energy so that at the classical level the mass of the monopole remains arbitrary. *A priori* there is nothing wrong with this, but nevertheless one may wonder whether one can have an analytic electroweak monopole which has a finite energy. The purpose of this Letter is to show that this is indeed possible, and to present analytic electroweak monopole and dyon solutions. Clearly the new monopoles should have important physical applications in the phenomenology of electroweak interaction.

Let us start with the Lagrangian of the standard Weinberg-Salam model,

$$\mathcal{L} = -|D_\mu \phi|^2 - \frac{\lambda}{2} \left(\phi^\dagger \phi - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{4} (F_{\mu\nu})^2 - \frac{1}{4} (G_{\mu\nu})^2,$$

$$D_\mu \phi = \left(\partial_\mu - i \frac{g}{2} \boldsymbol{\tau} \cdot \mathbf{A}_\mu - i \frac{g'}{2} B_\mu \right) \phi, \quad (1)$$

where ϕ is the Higgs doublet, $\mathbf{F}_{\mu\nu}$ and $G_{\mu\nu}$ are the gauge field strengths of $SU(2)$ and $U(1)$ with the potentials \mathbf{A}_μ and B_μ . Now we choose the following static spherically symmetric ansatz

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}} \rho(r) \xi(\theta, \varphi), \\ \xi &= i \begin{pmatrix} \sin(\theta/2) e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}, \quad \hat{\phi} = \xi^\dagger \boldsymbol{\tau} \xi = -\hat{r}, \\ \mathbf{A}_\mu &= \frac{1}{g} A(r) \hat{\phi} \partial_\mu t + \frac{1}{g} (f(r) - 1) \hat{\phi} \times \partial_\mu \hat{\phi}, \\ B_\mu &= -\frac{1}{g'} B(r) \partial_\mu t - \frac{1}{g'} (1 - \cos \theta) \partial_\mu \varphi, \end{aligned} \quad (2)$$

where (t, r, θ, φ) are the spherical coordinates. Notice that the apparent string singularity along the negative z -axis in ξ and B_μ is a pure gauge artifact which can easily be removed with a hypercharge $U(1)$ gauge transformation. Indeed one can easily excise the string by making the hypercharge $U(1)$ bundle non-trivial. So the above ansatz describes a most general spherically symmetric ansatz of a $SU(2) \times U(1)$ dyon. Here we emphasize the importance of the non-trivial $U(1)$ degrees of freedom to make the ansatz spherically symmetric. Without the extra $U(1)$ the Higgs doublet does not allow a spherically symmetric ansatz. This is because the spherical symmetry for the gauge field involves the embedding of the radial isotropy group $SO(2)$ into the gauge group that requires the Higgs field to be invariant under the $U(1)$ subgroup of $SU(2)$. This is possible with a Higgs triplet, but not with a Higgs doublet [3].

With the spherically symmetric ansatz (2) and with the proper boundary conditions one can obtain the Cho-Maison dyon solution whose magnetic charge is given by $4\pi/e$ [1]. The regular part of the solution looks very much like the Julia-Zee dyon solution [4], except that it has a non-trivial $B - A$ which represents the neutral Z boson content of the dyon solution. Of course the energy of the Cho-Maison solutions becomes infinite, due

to the magnetic singularity at the origin. A simple way to make the energy finite is to introduce the gravitational interaction [5]. But the gravitational interaction is not likely remove the singularity at the origin.

To construct the analytic monopole and dyon solutions, notice that a non-Abelian gauge theory in general is nothing but a special type of an Abelian gauge theory which has a well-defined set of charged vector fields as its source. This must be obvious, but this trivial observation reminds us the fact that the finite energy non-Abelian monopoles are really nothing but the Abelian monopoles whose singularity is regularized by the charged vector fields. From this perspective one can try to make the energy of the above solutions finite by introducing additional interactions and/or charged vector fields. In the following we present two ways to achieve this goal.

A) Electromagnetic Regularization

We could try to regularize the magnetic singularity of the Cho-Maison solutions with a judicious choice of an extra electromagnetic interaction of the charged vector field with the monopole. This regularization would provide a most economic way to make the energy of the Cho-Maison solution finite, because here we could use the already existing W boson without introducing a new source. To show that this is indeed possible notice that in the unitary gauge the Lagrangian (1) can be written as

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}(\partial_\mu \rho)^2 - \frac{\lambda}{2}\left(\frac{\rho^2}{2} - \frac{\mu^2}{\lambda}\right)^2 - \frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(G_{\mu\nu})^2 \\ & - \frac{1}{2}|D_\mu W_\nu - D_\nu W_\mu|^2 + \frac{1}{4}g^2(W_\mu^* W_\nu - W_\nu^* W_\mu)^2 \\ & + igF_{\mu\nu}W_\mu^* W_\nu - \frac{1}{4}\rho^2\left(g^2W_\mu^* W_\mu + \frac{1}{2}(g'B_\mu - gA_\mu)^2\right), \end{aligned} \quad (3)$$

where $W_\mu = \frac{1}{\sqrt{2}}(A_\mu^1 + iA_\mu^2)$, $A_\mu = A_\mu^3$, and $D_\mu W_\nu = (\partial_\mu + igA_\mu)W_\nu$. In this gauge the spherically symmetric ansatz (2) is written as

$$\begin{aligned} \rho &= \rho(r), \\ W_\mu &= \frac{i}{g} \frac{f(r)}{\sqrt{2}} e^{i\varphi} (\partial_\mu \theta + i \sin \theta \partial_\mu \varphi), \\ A_\mu &= -\frac{1}{g} A(r) \partial_\mu t - \frac{1}{g} (1 - \cos \theta) \partial_\mu \varphi, \\ B_\mu &= -\frac{1}{g'} B(r) \partial_\mu t - \frac{1}{g'} (1 - \cos \theta) \partial_\mu \varphi. \end{aligned} \quad (4)$$

To regularize the Cho-Maison dyon, we now introduce an extra interaction \mathcal{L}' ,

$$\mathcal{L}' = i\alpha g F_{\mu\nu} W_\mu^* W_\nu + \frac{\beta}{4} g^2 (W_\mu^* W_\nu - W_\nu^* W_\mu)^2. \quad (5)$$

With this additional interaction the energy of the dyon is given by $E = E_0 + E_1$, where

$$E_0 = \frac{2\pi}{g^2} \int_0^\infty dr \left\{ \frac{g^2}{r^2} + 1 - 2(1 + \alpha)f^2 + (1 + \beta)f^4 \right\},$$

$$\begin{aligned} E_1 = & \frac{4\pi}{g^2} \int_0^\infty dr \left\{ \frac{g^2}{2} (r\dot{\rho})^2 + \frac{g^2}{4} f^2 \rho^2 + \frac{g^2 r^2}{8} (B - A)^2 \rho^2 \right. \\ & + \frac{\lambda g^2 r^2}{2} \left(\frac{\rho^2}{2} - \frac{\mu^2}{\lambda} \right)^2 + (\dot{f})^2 + \frac{1}{2} (r\dot{A})^2 \\ & \left. + \frac{g^2}{2g'^2} (r\dot{B})^2 + f^2 A^2 \right\}. \end{aligned} \quad (6)$$

Clearly E_1 could be made finite with a proper boundary condition, but notice that when $\alpha = \beta = 0$, E_0 becomes infinite. To make E_0 finite we must require

$$1 + \frac{g^2}{g'^2} - 2(1 + \alpha)f^2(0) + (1 + \beta)f^4(0) = 0. \quad (7)$$

Furthermore to extremise the energy functional we must have

$$(1 + \alpha)f(0) - (1 + \beta)f^3(0) = 0. \quad (8)$$

Thus we must have

$$\begin{aligned} \frac{(1 + \alpha)^2}{1 + \beta} &= 1 + \frac{g^2}{g'^2} = \frac{1}{\sin^2 \theta_w}, \\ f(0) &= \frac{1}{\sqrt{(1 + \alpha) \sin^2 \theta_w}}, \end{aligned} \quad (9)$$

where θ_w is the Weinberg angle. In general $f(0)$ can assume an arbitrary value depending on the choice of α . But notice that, except for $f(0) = 1$, the $SU(2)$ gauge field is not well-defined at the origin. This means that only when $f(0) = 1$, or equivalently only when $\alpha = \beta$, the solution becomes analytic everywhere including the origin. So from now on we will assume $f(0) = 1$.

In this case the equations of motion that extremise the energy functional are given by

$$\begin{aligned} \ddot{f} - \frac{f^2 - 1}{\sin^2 \theta_w r^2} f &= \left(\frac{g^2}{4} \rho^2 - A^2 \right) f, \\ \ddot{\rho} + \frac{2}{r} \dot{\rho} - \frac{f^2}{2r^2} \rho &= -\frac{1}{4} (B - A)^2 \rho + \lambda \left(\frac{\rho^2}{2} - \frac{\mu^2}{\lambda} \right) \rho, \\ \ddot{A} + \frac{2}{r} \dot{A} - \frac{2f^2}{r^2} A &= \frac{g^2}{4} (A - B) \rho^2, \\ \ddot{B} + \frac{2}{r} \dot{B} &= \frac{g'^2}{4} (B - A) \rho^2, \end{aligned} \quad (10)$$

which can be integrated with the boundary conditions

$$\begin{aligned} f(0) &= 1, \quad A(0) = 0, \quad B(0) = b_0, \quad \rho(0) = 0, \\ f(\infty) &= 0, \quad A(\infty) = B(\infty), \quad \rho(\infty) = \rho_0 = \sqrt{2\mu^2/\lambda}. \end{aligned} \quad (11)$$

The result of the numerical integration for the finite energy dyon, together with the Cho-Maison dyon, is shown in Fig.1. It is really remarkable that *the finite energy solutions look almost identical to the Cho-Maison solutions, even though they no longer have the singularity at the origin and analytic everywhere.*

Clearly the energy of the above solutions must be of the order of the electroweak scale $M_W = g\rho_0/2$. Indeed for the monopole the energy with $\lambda/g^2 = 0.5$ is given by

$$E = 2.922 \sin^2 \theta_w \times \frac{4\pi}{e^2} M_W. \quad (12)$$

This demonstrates that the finite energy solutions are really nothing but the regularized Cho-Maison solutions which have a mass of the electroweak scale.

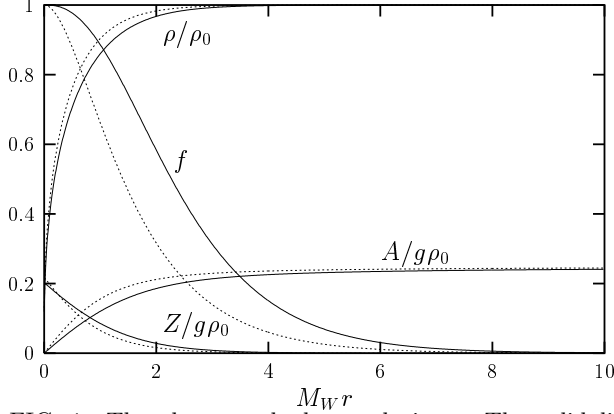


FIG. 1. The electroweak dyon solutions. The solid line represents the finite energy dyon and dotted line represents the Cho-Maison dyon, where $Z = B - A$ and we have chosen $\sin^2 \theta_w = 0.2325$, $\lambda/g^2 = 0.5$, and $A(\infty) = M_W/2$.

It is interesting to notice that for the monopole solution we can find the Bogomol'nyi type energy bound if we add an extra term $\delta\mathcal{L}$ in the Lagrangian (3)

$$\delta\mathcal{L} = -\left(\frac{1}{\sin^2 \theta_w} - \frac{1}{4}\right) g^2 \rho^2 W_\mu^* W_\mu. \quad (13)$$

Notice that this amounts to changing the mass of the W boson from $g\rho_0/2$ to $g\rho_0/\sin \theta_w$. In this case we can show that with ansatz (4) the energy of the monopole in the limit $\lambda = 0$ is bounded from below by

$$E \geq \frac{4\pi}{e} \rho(\infty) = \sin^2 \theta_w \frac{4\pi}{e^2} M_W. \quad (14)$$

Furthermore this bound is saturated by the following Bogomol'nyi type equation,

$$\begin{aligned} \dot{f} + \frac{e}{\sin^2 \theta_w} \rho f &= 0, \\ \dot{\rho} - \frac{1}{er^2} (1 - f^2) &= 0, \end{aligned} \quad (15)$$

which allows an analytic solution very much like the Prasad-Sommerfield solution. Notice that the energy of this solution has exactly the same form as the Prasad-Sommerfield monopole. Obviously the solution is stable since it is the lowest energy configuration.

B) Embedding to $SU(2) \times SU(2)$

As we have noticed the origin of the infinite energy of the Cho-Maison solutions was the magnetic singularity of

$U(1)_{\text{em}}$. On the other hand the ansatz (2) also suggests that this singularity really originates from the magnetic part of the hypercharge $U(1)$ field B_μ . So one could try to obtain a finite energy monopole solution by regularizing this hypercharge $U(1)$ singularity. This could be done by enlarging the hypercharge $U(1)$ and embedding it to another $SU(2)$. This, of course, is same as introducing a hypercharged vector field to the theory to regularize the $U(1)$ singularity.

To construct the desired solutions we introduce the hypercharge $SU(2)$ gauge field \mathbf{B}_μ and a scalar triplet Φ , and consider the following Lagrangian

$$\begin{aligned} \mathcal{L} = & -|D_\mu \phi|^2 - \frac{\lambda}{2} \left(\phi^\dagger \phi - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{4} (\mathbf{F}_{\mu\nu})^2 \\ & - \frac{1}{2} (\tilde{D}_\mu \Phi)^2 - \frac{\kappa}{4} \left(\Phi^2 - \frac{m^2}{\kappa} \right)^2 - \frac{1}{4} (\mathbf{G}_{\mu\nu})^2, \end{aligned} \quad (16)$$

where $\tilde{D}_\mu \Phi = (\partial_\mu + g' \mathbf{B}_\mu \times) \Phi$. Now in the unitary gauge let $X_\mu = (B_\mu^1 + i B_\mu^2)/\sqrt{2}$, $B_\mu = B_\mu^3$, $\tilde{D}_\mu X_\nu = (\partial_\mu + i g' B_\mu) X_\nu$, $\Phi = (0, 0, \sigma)$, and choose the static spherically symmetric ansatz

$$\begin{aligned} \sigma &= \sigma(r), \\ X_\mu &= \frac{i}{g'} \frac{h(r)}{\sqrt{2}} e^{i\varphi} (\partial_\mu \theta + i \sin \theta \partial_\mu \varphi), \end{aligned} \quad (17)$$

together with (4). With this we obtain the following equations,

$$\begin{aligned} \ddot{f} - \frac{f^2 - 1}{r^2} f &= \left(\frac{g^2}{4} \rho^2 - A^2 \right) f, \\ \ddot{\rho} + \frac{2}{r} \dot{\rho} - \frac{f^2}{2r^2} \rho &= -\frac{1}{4} (B - A)^2 \rho + \lambda \left(\frac{\rho^2}{2} - \frac{\mu^2}{\lambda} \right) \rho, \\ \ddot{A} + \frac{2}{r} \dot{A} - \frac{2f^2}{r^2} A &= \frac{g^2}{4} \rho^2 (A - B), \\ \ddot{h} - \frac{h^2 - 1}{r^2} h &= (g'^2 \sigma^2 - B^2) h, \\ \ddot{\sigma} + \frac{2}{r} \dot{\sigma} - \frac{2h^2}{r^2} \sigma &= \kappa \left(\sigma^2 - \frac{m^2}{\kappa} \right) \sigma, \\ \ddot{B} + \frac{2}{r} \dot{B} - \frac{2h^2}{r^2} B &= \frac{g'^2}{4} \rho^2 (B - A). \end{aligned} \quad (18)$$

Now with the boundary condition (11) and with

$$\begin{aligned} h(0) &= 1, \quad \sigma(0) = 0, \\ h(\infty) &= 0, \quad \sigma(\infty) = \sigma_0 = \sqrt{m^2/\kappa}, \end{aligned} \quad (19)$$

one may try to find the desired solution. Clearly the spontaneous symmetry breaking of the hypercharge $SU(2)$ at the infinity adds a new scale $M_X = g'\sigma_0$, an intermediate scale which lies somewhere between the grand unification scale and the electroweak scale, to the theory. Now, consider the monopole solution and let $A = B = 0$ for simplicity. Then in the limit $\lambda = \kappa = 0$ we obtain the solution shown in Fig.2 with $M_X = 10M_W$, whose energy is given by

$$E = (\cos^2 \theta_w + 0.195 \sin^2 \theta_w) \frac{4\pi}{e^2} M_X. \quad (20)$$

Clearly the solution describes the Cho-Maison monopole whose singularity is regularized by the hypercharge vector field X_μ .

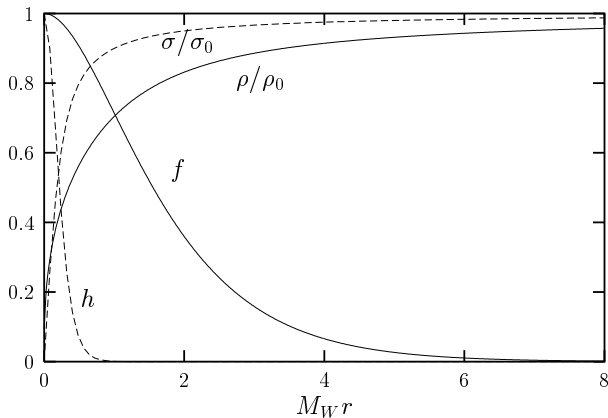


FIG. 2. The $SU(2) \times SU(2)$ monopole solution.

It has generally been assumed that the finite energy monopoles could exist only at the grand unification scale [6]. But our result tells that *there may exist a totally new class of electroweak monopole and dyon whose mass is much smaller than the monopoles of the grand unification*. Obviously the electroweak monopoles are topological solitons which must be stable.

Strictly speaking the finite energy solutions are not the solutions of the Weinberg-Salam model, because their existence requires a generalization of the model. But from the physical point of view there is no doubt that they should be interpreted as the electroweak monopole and dyon, because they are really nothing but the regularized Cho-Maison solutions whose size is fixed at the electroweak scale. In spite of the fact that the Cho-Maison solutions are obviously the solutions of the Weinberg-Salam model one could try to object them as the electroweak dyons, under the presumption that the energy of the solutions could be made finite only at the grand unification scale. Our work shows that this objection is groundless, and assures that it is not necessary for us to go to the grand unification scale to make the energy of the Cho-Maison solutions finite.

We close with the following remarks:

1) The electromagnetic regularization of the Dirac monopole with the charged vector fields is nothing new. In fact it was this regularization which made the energy of the 't Hooft-Polyakov monopole finite. Furthermore it has been known that the 't Hooft-Polyakov solution is the only analytic solution (with $\alpha = \beta = 0$) which one could obtain with this technique [7]. What we have shown in this paper is that the same technique also works to regularize the Cho-Maison solutions, but with nonvanishing α and β .

2) The introduction of the additional interactions (5) and (13) to the Lagrangian (1) could spoil the renormalizability of the Weinberg-Salam model (although this issue has to be examined in more detail). How serious would this offense, however, is not clear at this moment. Here we simply notice that the introduction of a non-renormalizable interaction (like a gravitational interaction) has been an acceptable practice to study finite energy classical solutions.

3) The embedding of the electroweak $SU(2) \times U(1)$ to a larger $SU(2) \times SU(2)$ or $SU(2) \times SU(2) \times U(1)$ could naturally arise in the left-right symmetric grand unification models, in particular in the $SO(10)$ grand unification, although the embedding of the hypercharge $U(1)$ to a compact $SU(2)$ could turn out to be too simple to be realistic. Independent of the details, however, our discussion strongly suggests that the electroweak monopoles at the intermediate scale M_X could be possible in a realistic grand unification.

Certainly the existence of the finite energy electroweak monopoles should have important physical implications. Probably they could be the only finite energy topological monopoles that one could ever hope to produce with the (future) accelerators. A more detailed discussion of our work will be published in a separate paper [8].

Acknowledgments

The work is supported in part by the Ministry of Education and by the Korean Science and Engineering Foundation.

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- [1] Y.M. Cho and D. Maison, Phys. Lett. **B391**, 360 (1997).
 - [2] Yisong Yang, Proc. Roy. Soc. **A** (1997), in press.
 - [3] P. Forgács and N.S. Manton, Commun. Math. Phys. **72**, 15 (1980).
 - [4] B. Julia and A. Zee, Phys. Rev. **D11**, 2227 (1975); M.K. Prasad and C.M. Sommerfield, Phys. Rev. Lett. **35**, 760 (1975).
 - [5] F.A. Bais and R.J. Russell, Phys. Rev. **D11**, 2692 (1975); Y.M. Cho and P.G.O. Freund, Phys. Rev. **D12**, 1711 (1975); P. Breitenlohner, P. Forgacs, and D. Maison, Nucl. Phys. **B383**, 357 (1992).
 - [6] C.P. Dokos and T.N. Tomaras, Phys. Rev. **D21**, 2940 (1980); Y.M. Cho, Phys. Rev. Lett. **44**, 1115 (1980).
 - [7] K. Lee and E. Weinberg, Phys. Rev. Lett. **73**, 1203 (1994); C. Lee and P. Yi, Phys. Lett. **B348**, 100 (1995).
 - [8] Y.M. Cho and K. Kimm, APCTP-97-12, to be published.